Quantifying the Waves Generated by Vessels Operating in Sheltered Waterways

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Abstract
The waves generated by vessels (often referred to as wave wake, or simply wash) that operate within sheltered waterways can result in a variety of issues for other users of the waterway and the surrounding environment. This has resulted in a growing need for tools that accurately predict the characteristics of these waves to assist in the provision of effective waterways management.

This is particularly the case when the combination of the vessel speed and water depth beneath the vessel results in depth Froude numbers in the trans-critical region (typically $0.75 < Fr_h < 1.0$). This scenario is investigated for multiple commercial passenger catamaran ferries (with waterline lengths around 33 to 36 m) through comparisons of full-scale wave wake trials data against predictions from a validated empirical tool. The comparisons confirm that the empirical tool produces accurate predictions for most cases, including both deep and shallow water over a wide range of vessel speeds, however it can significantly underpredict the excessively high energy waves created during the trials at speeds close to the critical depth Froude number (Fr_h ~1.0).

Further comparisons between full scale trials data and empirical predictions are presented for five waterski boats. In this case the good agreement has allowed the trials data to be used to enhance the predictive capabilities of the empirical tool by expanding the limit of applicability to cover significantly higher speeds.

The implications of an inadequate assessment of vessel operations at trans-critical speeds are also discussed. It is recommended that the first step of any such assessment include two simple “back-of-the-envelope” calculations. If either of these quick checks fall within regions of concern, then further investigation is warranted to avoid unwanted issues.

Keywords: wave wake; wash; predictions; ferry operations; catamaran; full-scale experiments.

1. Introduction

There is a demonstrated need to understand the phenomenon of vessel wave wake due to the many issues that waves generated by marine vessels have caused for other users of waterways and the surrounding environment, such as shoreline erosion, damage to marine structures and even some fatalities. For example, see PIANC (2003); Parnell and Kofoed-Hansen (2001); MAIB (2000). This has led to the demand for predictive tools that can quantify the characteristics of the waves generated by marine craft so a reliable assessment of their likely effect can be performed during the early stages when designing new vessels and waterway infrastructure. Ideally, such tools should be developed and validated to predict the waves generated by any typical hull form under all practical operational conditions.

The complex array of variables involved can make the development of wave wake prediction tools a difficult task, particularly when attempting to accurately predict the effects of water depth and the far-field. Propagating wave phenomena such as dispersion and attenuation can be challenging to estimate, Campana *et al*. (2005 and 2008); Macfarlane (2012). Many of the problems associated with vessel-generated waves occur in shallow and/or restricted water where the characteristics and pattern of waves generated is very different to that generated in deep water, Cox (2020). The main factors to consider include the:

- characteristics of the vessel (speed, waterline length, displacement, hull form, beam, draught, etc),
- characteristics of the waterway (water depth, bathymetry, width, shoreline details),
- sailing line of the vessel within the waterway; and,

rate of decay of the generated waves.

2. Vessel Wave Wake

2.1 The Effect of Water Depth

The wave pattern generated by a vessel is largely independent of vessel form, but is greatly affected by water depth and vessel speed, Cox and Macfarlane (2019). Traditionally, naval architects and maritime engineers have adopted the length Froude number, Fr_L, as defined in Equation 1, to non-dimensionalise vessel speed.

$$Fr_L = \frac{u}{\sqrt{gL}} \qquad (1)$$

However, in cases where there is finite water depth the defining parameter is not the length Froude number, but the depth Froude number, Fr_h, a non-dimensional relationship between vessel speed and water depth beneath the vessel, as defined in Equation 2:

$$Fr_h = \frac{u}{\sqrt{g\mathrm{h}}} \quad (2)$$

The significant effect that water depth has on vessel wave wake makes it useful to refer to some very distinct speed-related categories. At a vessel speed below depth Froude number of one, the speed is said to be sub-critical. A depth Froude number of 1.0 is termed the critical speed and speeds leading up to or close to the critical speed are sometimes referred to as trans-critical speeds (approximately $0.75 \leq Fr_h \leq 1.0$). The value of the lower and upper bounds of the trans-critical range can vary between reference texts on the subject. Speeds above a depth Froude number of 1.0 are super-critical. Simplified sketches indicating how these complex wave patterns alter in each of these categories are provided in Figure 1.

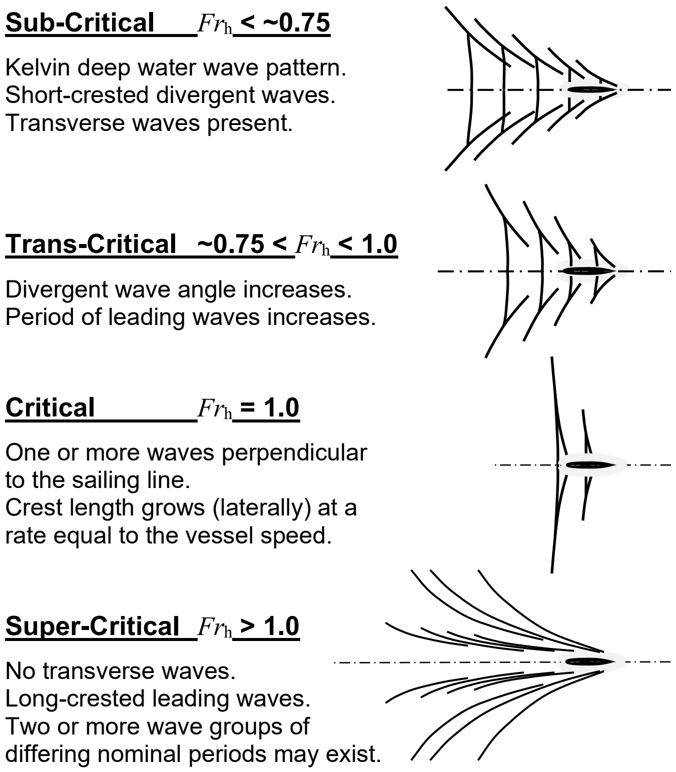

Figure 1 Simplified depictions of wave wake patterns for each vessel speed regime.

2.2 Quantifying Wave Wake

During the 1990s, it was common to describe these complex wave patterns by identifying and quantifying just a single 'maximum' wave within the entire wave train, usually the highest, Stumbo et al. (1999). It is believed that this approach was taken due to the limited understanding of the science at that time and a preference by regulatory authorities to use measures that could be easily visualised. Not surprisingly, it has been proven that such a simplistic approach is very often inadequate when attempting to assess marine craft, particularly when operating at trans-critical or super-critical speeds, Macfarlane (2012). It was shown that at least two, but preferably three, key waves must be identified in order to undertake even the most basic assessment of the potential effects of waves created by marine vessels. These waves, which represent groups of waves having similar period, are defined as follows (refer to Macfarlane (2012) for a more detailed description):

- Wave A – the leading diverging wave, which is the wave that will possess the longest period.
- Wave B – the most significant wave following the leading wave (Wave A). The period will be shorter than the leading wave, but often not by a large margin, whereas the height is very often greater than the leading wave.
- Wave C – it is common for a group of short period divergent waves to be generated and Wave C is defined as being the highest wave within this group. This wave always follows Waves A and B, hence will possess the shortest wave period of these three key waves.

As an example, Waves A, B and C are identified in Figure 2, which shows a typical time series plot of the waves generated by a ship model travelling at a super-critical speed. As can be seen, the period of each wave 'group' changes significantly.

As a general guide, depth Froude number has its greatest effect when the water depth is less than about one-quarter the vessel's (dynamic) waterline length ($h/L < 0.25$); it has moderate influence at depths up to one-half the waterline length ($h/L < 0.5$) and has little influence at depths greater than the waterline length ($h/L > 1$), Cox and Macfarlane (2019).

Over the past two decades, in addition to quantifying the wave height and period of key wave(s), many wave wake studies and regulatory criteria have been based on the energy (E) in each wavelength (per unit width of wave crest), commonly defined by Equation 3, Parnell and Kofoed-Hansen (2001); Stumbo et al. (1999); Macfarlane (2002):

$$E = \frac{\rho g^2 H^2 T^2}{16\pi} \quad (3)$$

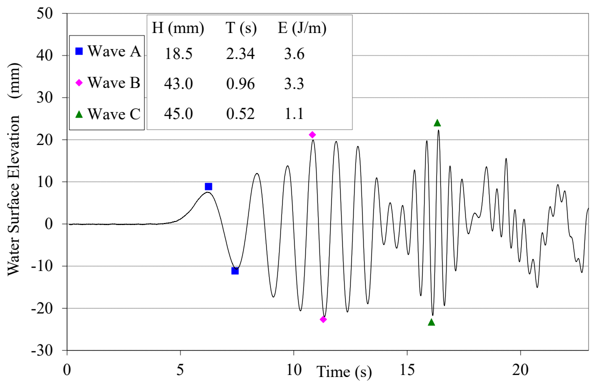

Figure 2 Example of a (model scale) time series plot of a longitudinal cut of a vessels wave wake indicating three different wave 'groups'. The height, period and resultant energy of each wave type is indicated. In this example, the wave with the lowest height (Wave A) has the greatest energy – regulatory criteria that focus solely on the highest wave would assess Wave C (only 1/3 the energy).

The author has developed an empirical tool that can provide very rapid wave wake predictions, Macfarlane (2012); Macfarlane et al. (2014). Based on the analysis from a comprehensive series of model scale experiments on a wide range of hull types, water depths and vessel speeds, the tool can predict the characteristics of all three key waves (A, B and C) for sub-, trans- and super-critical speeds. Predictions of the four key variables of wave height (via the constant, γ), wave period (T), wave decay rate (n) and wave angle (θ) for all three waves of interest are calculated based on several principal vessel and environment details.

This empirical prediction tool, termed the *Wave Wake Predictor*, was first developed between 1996 and 2000 from an extensive series of physical scale model experiments within a deep and wide test basin on more than 80 different hull form configurations (involving over 6000 individual wave cuts). This early version of the tool focussed purely on deep water conditions (sub-critical depth Froude numbers). In 2000, a wide basin that was ideally suited to investigate the effects from shallow water operation was commissioned at AMC (2023a). This controlled environment hydrodynamic facility led to an even more ambitious experimental campaign (involving a further ~8,000 wave cuts), plus the more considered approach to the analysis of multiple waves (A, B and C) within each wave packet: something that was essential when considering waves generated by vessels over the full range of practical speed/depth zones (sub-critical, trans-critical and super-critical depth Froude numbers).

By 2012 the 'next-generation' *Wave Wake Predictor* had been developed and successfully validated against the full scale trials data for over six different vessels. Since then, the tool has been continually developed to further expand its capabilities in terms of (a) the range of applicable vessel types and sizes, including 'extremes' such as wake boats for wake-surfing and other water recreational craft, and (b) estimating the effect that other relevant factors have on the characteristics of the waves generated, such as (but not limited to): the effect when a vessel accelerates or decelerates; the effect of narrow river or shipping channels (lateral banks); the effect of varying bathymetry; and the effect of a manoeuvring (turning) vessel.

The base version of the *Wave Wake Predictor* can be accessed online at AMC (2023b). It has been comprehensively validated through direct comparison against wave wake data collected from many series of full scale trials conducted on various different types of hull form on several different sheltered waterways. Some of these comparisons are covered in more detail in Macfarlane (2012). However, as highlighted later in this paper, accurately predicting the characteristics of vessel generated waves close to the critical speed can be very difficult – and, unfortunately, this can coincide with the conditions where the most energetic (and potentially damaging) waves are generated.

3. Full Scale Trials and Empirical Predictions

The process of validating the *Wave Wake Predictor* has involved dedicated, professionally executed full scale trials on more than 25 different marine craft to date. The range of vessels includes, but is not limited to, personal water craft (jet skis), recreational craft, commercial fishing boats and passenger ferries.

The success of field trials is highly dependent on the adoption of rigorous and time-proven testing methodology, instrumentation and analysis procedures. Vessel wave wake is not a steady-state phenomenon (from a fixed reference frame) and its assessment is reliant on consistency. The testing methodology adopted for all trials used to validate the *Wave Wake Predictor* ensured that the results were not site-specific and can be transposed with other results from other sites. Full-scale experiments are often subjected to many natural and procedural influences that affect the accuracy of the results. Quite besides complications such as wind waves, currents, and variable water depths, other influences must be tempered to improve accuracy and repeatability. The most important issues are discussed in more detail by Macfarlane (2012).

For the present study, full scale experiments were performed for several typical catamaran passenger ferries used for urban transport with lengths ranging from 25 to 36 m. A focus of these trials was to investigate a range of shallow water depths ranging from 12 m down to 3.5 m. The results were found to clearly demonstrate some of the challenges faced when vessels operate in shallow water. In this section, data acquired from these on-site trials are compared against predictions from the *Wave Wake Predictor*. Two cases involving different passenger catamarans have been selected for presentation and discussion in this paper. A third case extends the study by investigating the waves generated by a number of recreational ski boats.

Case 1: Catamaran

length = 36 m
displacement = 64 t
slenderness ratio = 9.0
water depth = 12 m
lateral distance = 60 m

In this case, where h/L = 0.33, good agreement is found when the full scale trials data is directly compared against predictions from the *Wave Wake Predictor*. The results for wave height, wave period and wave energy (calculated using Equation 3) are each presented as a function of depth Froude number in Figures 3, 4 and 5 respectively. At this (mostly) deep water depth, it can be seen that the empirical predictions of wave height, period and energy all compare favourably with the trials data at all speeds, although there are indications that the tool may under predict at the highest Fr_h investigated in the full scale trials (which are just entering the trans-critical zone). In this specific case, where virtually all speeds are sub-critical, it is only Wave B that is compared in the results presented (for clarity) as their heights and energies are more significant than Waves A and C.

Further full scale experiments were performed on multiple catamaran ferries in the shallower water depth of 8 m (with h/L of approximately 0.22 to 0.24) with the empirical predictions showing similarly good agreement (not presented here). It is concluded that the empirical prediction tool provides a good engineering approximation of the likely wave characteristics for both the deep and intermediate water depth conditions.

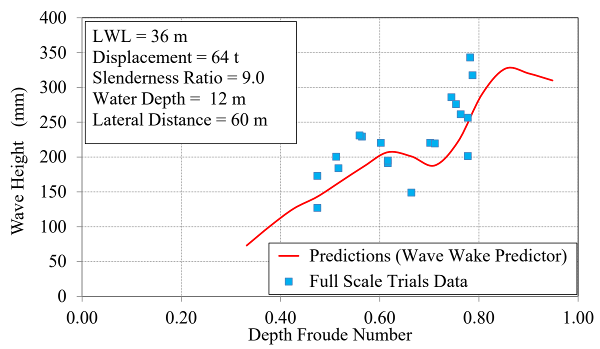

Figure 3 Case 1: Wave height as a function of depth Froude number: SR = 9.0, h = 12 m. Deep water case, where all vessel speeds are sub-critical. The blue data points are the measurements from the full scale experiments, the red curve is the empirical predictions. Favourable correlation is observed between the empirical predictions and full scale trials data.

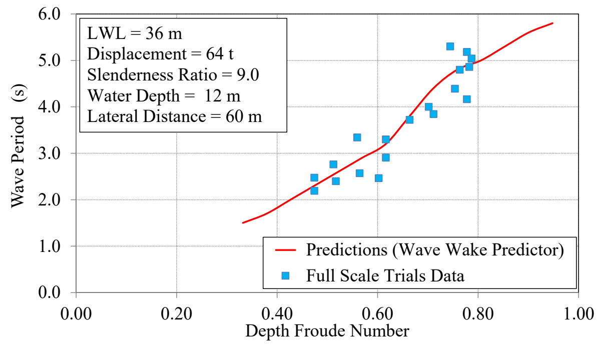

Figure 4 Case 1: Wave period as a function of depth Froude number: SR = 9.0, h = 12 m. As for wave height, good correlation is also observed for wave period.

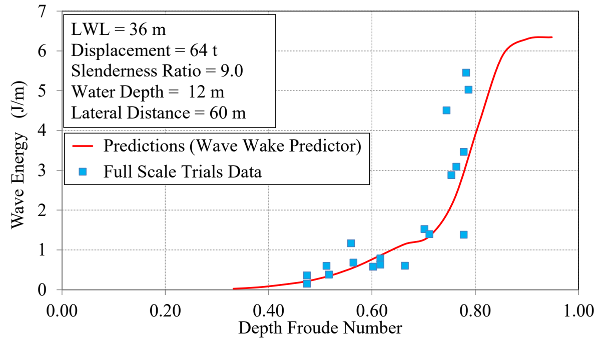

Figure 5 Case 1: Wave energy as a function of depth Froude number: SR = 9.0, h = 12 m. As wave energy is equally a function of wave height and period (refer Equation 3), both of which compared favourably (Figures 3 and 4), a similar result is observed for wave energy.

Case 2: Catamaran

length = 33 m
displacement = 86 t
slenderness ratio = 7.5
water depth = 3.5 to 6 m
lateral distance = 60 m

The significant effect due to limited water depth is highlighted in Case 2, involving a shorter, heavier catamaran. In an ideal world, all key variables should remain constant for both the full scale trials and empirical predictions to allow direct comparison between data sets, but, in this case this was not possible. However, this situation is beneficial as it highlights two key findings: (a) how dramatically the wave characteristics can alter in shallow water, and (b) how easy it is to be misled by predictions that may otherwise be considered realistic.

The only difference between the two data sets presented in Case 2 is a seemingly small variation in water depth: the full scale trials were performed in approximately 3.5 m deep water (h/L = 0.11) while the predictions from the *Wave Wake Predictor* are provided for the slightly deeper depth of 6.0 m (h/L = 0.18). This h/L value was the lower limit of applicability for the *Wave Wake Predictor* for a vessel of this length at the time of the full scale trials.

The wave height, period and energy results from the full scale trials and empirical predictions are compared in Figures 6, 7 and 8. Firstly, there are two clear differences between the data presented for Cases 1 and 2: here the speed range covers all four zones (sub-, trans-, critical and super-critical speeds); and, Waves A and B must both be considered as they are equally significant (the height of Wave C was also often significant, but due to their lower period – hence also energy – they have not been presented to improve clarity).

Despite the difference in water depth, the agreement between the trials data and empirical predictions is generally very good for all three quantities for both sub-critical speeds (Fr_h < ~0.75) and most super-critical speeds (typically in excess of Fr_h ~1.2). However, some very significant differences are observed within the trans-critical speeds where the reduced water depth has resulted in the generation of large, long-period waves as the critical speed is approached. For example, between 0.9 < Fr_h < 1.05 both wave height and period (Figures 6 and 7) can be under predicted by as much as ~300%. As can be seen from Equation 3, wave energy is proportional to the square of both wave height and period, thus these differences are amplified such that the wave energy during the trials is approximately 20 times greater than the predictions within this narrow range of speeds.

At this point it is worth recalling that the *Wave Wake Predictor* was developed specifically to deal with effects of water depth and has been validated through multiple comparisons with full scale trials data, however the massive differences found from this seemingly small reduction in water depth (6.0 to 3.5 m) - beyond the prediction tool's limit of applicability - emphasizes the high level of precision and awareness that is required when dealing with shallow water situations close to the critical speed. As a result, it is recommended that extreme care be taken when attempting to predict wave wake from vessels operating in shallow water and that it is good practice to validate all predictions, especially when h/L < 0.5, and is virtually essential in more extreme cases when h/L < 0.2. Where possible, it is recommended that reliable model and/or full scale data be used or acquired for this purpose.

The author is aware of studies where the effect of water depth has been ignored and assessments have been based purely on simplistic deep water predictions (also often using unvalidated numerical/ analytical methods). Figure 9 highlights the dangers of such an approach, by including predictions of wave energy for the <u>deep water</u> case (h/L > 1.0) to directly compare against the shallow water results previously shown in Figure 8. In this case where the water depth varies widely between data series, it is often more appropriate to compare results using the length Froude number. As can be seen, the comparison between the reality in shallow water (full scale trials data) and the deep water predictions (dashed curves) is even more stark.

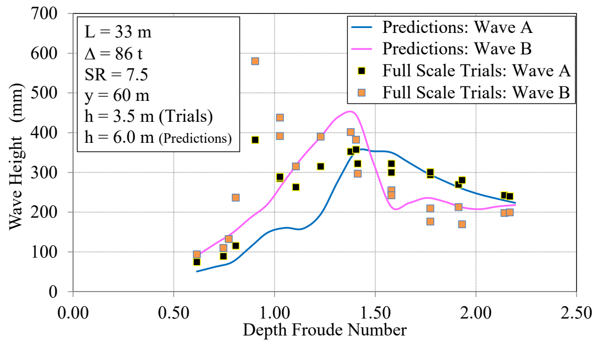

Figure 6 Case 2: Wave height as a function of depth Froude number: SR = 7.5, h = 3.5 m (Trials) and h = 6.0 m (Predictions)

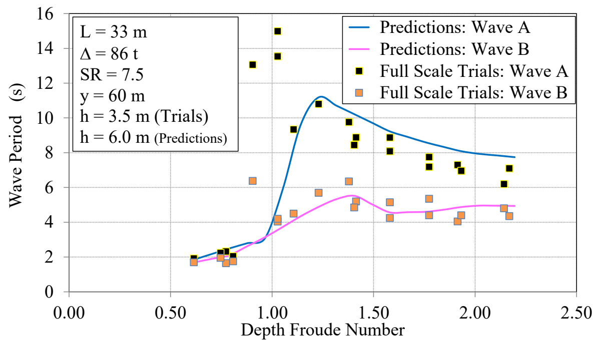

Figure 7 Case 2: Wave period as a function of depth Froude number: SR = 7.5, h = 3.5 m (Trials) and h = 6.0 m (Predictions)

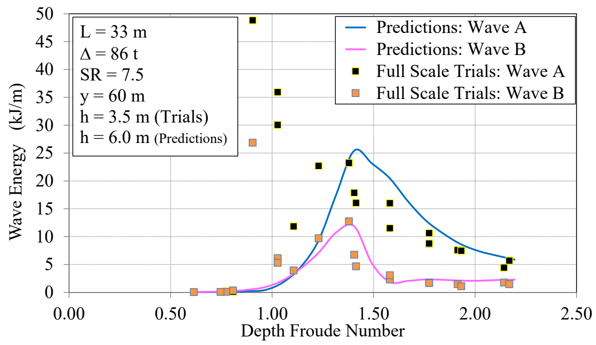

Figure 8 Case 2: Wave energy as a function of depth Froude number: SR = 7.5, h = 3.5 m (Trials) and h = 6.0 m (Predictions). Note the significant increase in wave energy from the full scale trials close Fr_h of unity – a result of the higher height and longer period seen in Figures 6 and 7 respectively.

The obvious conclusion that can be drawn from the results presented in Figures 6 to 9 is that operation of such vessels close to the critical speed at low values of h/L should be avoided as much as practical. In addition to the potential wave wake issues, there will be an increase in fuel consumption – most of which will be used to generate these large waves. In many cases it is very easy to identify the worst conditions to avoid – all it requires is a simple calculation of the depth Froude number (Equation 2) and the water depth to vessel length ratio (h/L).

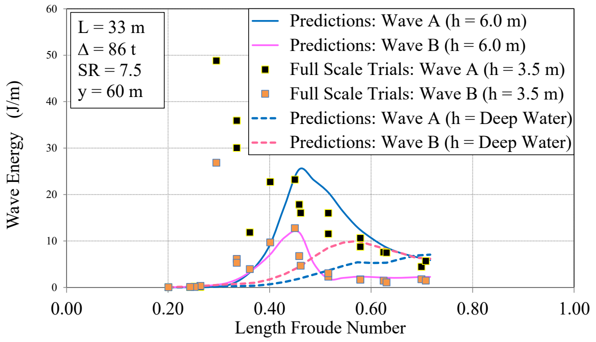

Figure 9 Case 2: Wave energy as a function of length Froude number: SR = 7.5, h = 3.5 m (Trials) and h = 6.0 m (Predictions) and deep water (Predictions). The difference between the full scale trials and deep water predictions (dashed curves) is stark.

Cox & Macfarlane (2019) present a figure designed to help identify high risk zones using these simple ratios. Their figure graphically outlines the relationship between h/L and Fr_L and delineates operation according to wave wake risk; seeking to avoid speeds where adverse vessel and depth dynamics combine.

The capabilities and validity of the *Wave Wake Predictor* are under continual development. The data from this complete series of full scale trials, of which about 20% is presented in this paper, have been used to further validate the tool for a wider range of hull forms, water depths and speeds. The outcomes of the present study have highlighted the need to extend the limit of applicability towards more extreme shallow water depths and significant progress towards this goal has since been made. This required the conduct of additional systematic model scale experiments in a controlled environment and associated analysis.

Case 3: Ski Boats
length = 5.4 to 6.3 m
displacement = 1.1 to 1.6 t
slenderness ratio = 4 to 5
water depth = 2.5 to 4.1 m

Other limits of applicability of the *Wave Wake Predictor* can and have been expanded using different approaches. As an example, the maximum speed that can be predicted was originally determined by the limits of the hydrodynamic facility that the model tests were performed. For small high speed hulls, such as ski boats, the maximum length Froude number that can be achieved on a suitably scaled model (Fr_L ~1.35) is considerably lower than many of these craft commonly operate (Fr_L > 2). High-quality full scale trials data from ten different ski boats has been used to significantly extend the upper limit of applicability of the prediction tool to Fr_L = 2.2. This is demonstrated in Figure 10, where the solid curve represents the predictions of wave period from the base version of the predictor and the red dashed curve the improved version. The results from the full scale experiments on five of the ski boats are shown as data points. Further results for the quantities of wave height and energy are presented in Macfarlane (2012). Similar improvements have been implemented within the *Wave Wake Predictor* using other relevant model and full scale data.

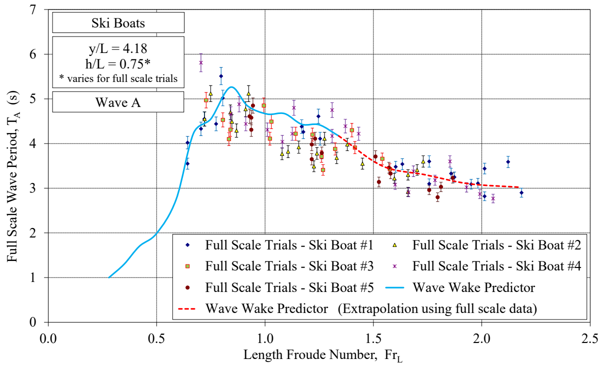

Figure 10 Results of full scale trials and predictions of wave periods generated by multiple ski boats as a function of length Froude number. This highlights the *Wave Wake Predictor's* increased range of speeds through the use of reliable full scale trials data. The range of speeds that the tool could originally predict is depicted by the blue curve, and following enhancement in red.

Experimental Uncertainty

As expected, experimental scatter is present in the full scale trials data presented in this paper. Even though much effort has gone into maintaining consistency during the conduct of the trials, the large number of variables and practical realities ensure that there will always be a greater degree of scatter in results from experiments conducted in any uncontrolled environment compared to those performed in the controlled environment provided by specialist hydrodynamic facilities, such as the AMC shallow water wave basin.

From the Author's experience, the level of variability displayed in Figures 3 to 10 are considered relatively small for full scale wave wake data. It is important to recognize that notably greater variation would certainly occur during normal vessel operations, suggesting that a suitable factor of safety should be considered when assessing acceptable regulatory limits.

The accuracy of the measurements of both wave height and period is estimated to be within +/-5%. This does not entirely account for variations in vessel speed, water depth, lateral distance or environmental influences such as wind waves and currents, all of which may result in increased data scatter. It is estimated that the combined effect of measurement accuracy and these uncontrolled sources of potential error may account for up to 10% variation in the experimental results.

4. Conclusions

This paper presents a general introduction to the basic science behind vessel generated waves, a brief description on the development of the *Wave Wake Predictor,* and results from full scale wave wake trials. This information has been used to highlight some of the challenges faced when considering vessel operations in shallow water, which is common to many sheltered waterways.

It was confirmed that it is best to assess representative waves for at least two wave groups (the leading long period wave and the most energetic wave, which is often the highest) for any vessel operations that involve speeds in either trans-critical, critical or super-critical depth Froude numbers.

The trials data displayed highly non-linear behaviour when a vessel operates in shallow water close to the critical speed. As a result, it is strongly recommended that any methods used to predict wave wake for shallow water vessel operations ($h/L < 0.5$) be appropriately validated to accurately account for the specific conditions under consideration. If such validation can't be demonstrated then it is recommended that either model or full scale experiments be performed, particularly where $h/L < 0.25$. Even a very limited experimental investigation should provide an indication of the likely wave characteristics that may be generated. Care should be taken into the specific combinations of speed and water depth as extreme conditions are likely to be concentrated within a narrow range.

In any study of this nature, it is highly recommended that the first step include the very simple "back-of-the-envelope" calculations of the depth Froude number (Equation 2) and the water depth to vessel length ratio (h/L). If either of these quantities fall within the regions of concern raised in this paper, then further investigation is certainly warranted to avoid the unwanted issues for other users of the waterway and the surrounding environment.

5. Acknowledgements

All full scale vessel trials such as those referred to in this report require considerable effort and input from many individuals and organizations to ensure their success. The efforts of the individuals and support of the organizations involved are gratefully acknowledged as their cooperation was crucial to the campaign.

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